Gestures in the blackboard work of mathematics instruction[1]

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Abstract. Lectures in mathematics consists almost entirely of the lecturer writing definitions, theorems, and proofs on the blackboard (often reproducing word-by-word what is distributed in advance in lecture notes) while simultaneously commenting on what is being written. The writing, talking, and gesturing conjointly formulate the cohesive logic of the mathematical argument that the formulae instantiate. In the first part we examine the *blackboard organization of the exposition*: what is written is not just written 'anywhere', but the physical structure of the blackboard is organized into segregated fields so as to re-order the formulae on the board in a way that displays their mathematical role amongst the interrelated constituents of the mathematical argument put forward. The second part focuses on how gestures are used *in conjunction with and coordination of* what is being written on the blackboard and what is being said.

Keywords. Mathematics; scientific practice; ethnomethodology; workplace studies.

1. Introduction

Eric Livingston, in his book *The Ethnomethodological Foundations of Mathematics* (1986), argues that the heart of mathematical work is at and around the blackboard. Like Livingston, we give attention to cases in which established proofs are 'gone through again'. We look at lectures to master level students in mathematical logic and we will examine the lecturer's extensive gestural work in *setting out* these proofs. These gestures often acquire their definite sense from the mathematical structures that are under exposition, and it is these features to which we will pay particular attention.

In terms of our presence at this conference on gesture, it is because of the notable character of the lecturer's gesturing as embedded in the 'multi-tracked' environment of the proofs as written out in the lecture notes that students can read, in the writing out of the formulae on the boards and of the recitation and commentary upon the written out proofs that accompanies the writing out of the proofs on the board. Thus we can build on McNeill's (1979, 1992) early work on gestures of mathematicians, in which he demonstrated that gestural work is embedded in the organisation of the spoken commentary – and thus both articulated with, and used to capture, the progression of the course of the mathematical reasoning under construction.

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Despite many claims about the capacity of a sociology of scientific knowledge to give an account of mathematics' 'content', we feel few actual attempts are made to do this. Those that do undertake such investigations make less effort to analyse the nature of 'the mathematical content', than they do to 'prove' sociological points about it. Consequently, such studies largely depend on relatively impoverished versions of mathematical materials and practices.

Given such a dissatisfaction, it is natural enough to want to take a look at the mathematical content in a situation where many of its professionally relevant features will be on display, namely in classroom instruction to those who are at the beginning of a mathematical career. Our initial observational materials are records of mathematical lectures. Here, we are interested in the way that mathematical organisation is laid out. The lecturers involve the presentation of proofs, and we will be looking at the presentations to see how the *mathematical* organisation of the proof is provided in the lecture, what kinds of constituents comprise a proof, how are they are assembled into the proof's development and, eventually, overall construction, what are the operational structures of the mathematical reasoning in setting out the task(s), electing 'next steps', providing for the demonstrability of the 'moves', making the interdependencies between parts of the proof perspicuous, and identifying the 'deliverables' of any particular instructional achievement.

Mathematical lectures have the distinctive feature that the students are provided, in advance, with lecture notes in which the proofs to be 'gone over' in the lectures are fully spelled out. Nonetheless those same proofs will be written out on the blackboard as the course progresses. The question then seems to be: What is the 'point' of lectures in mathematics – if the 'content' is already available? Getting an answer to that question would, we think, be a first step towards understanding the 'quiddity' of mathematical work.

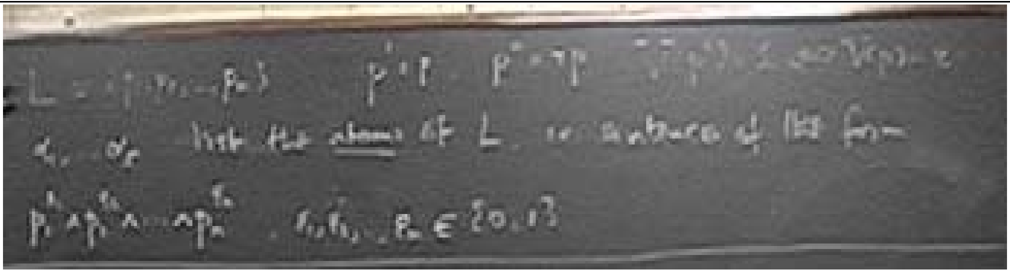
2. Fragment One

We will discuss two short extracts from lectures given by two different lecturers.

Our first excerpt is taken from the beginning of a lecture (see the transcript in Appendix I). *Before* the start of the lecture the lecturer has already written a 'summary' of what they did in the previous lecture on the board. This summary is depicted in Figure 1 (first as an enlarged snapshot from the videotape and then typed out for easy readability). The episode exhibits how the lecturer goes over the formulae that he has already written out and thereby resumes what was accomplished in the previous lecture.

We look at this excerpt mainly to identify some of the diversity of gestures in it. In particular, we look at how the form of some of these gestures depends upon the ways in which they iconicise some of the mathematical operations involved in the proof. We also note the way in which gestures are used to 'bring out' features of the structure and rationale of the proof that are not perhaps immediately apparent (at least to relative beginners).

Given the short time for this presentation, we can only mention a few features of this fragment.



$$L = \{p_1, \ldots, p_n\} \qquad p^1 = p,\ p^0 = \neg p \qquad V(p^{\varepsilon}) = 1 \Leftrightarrow V(p) = \varepsilon$$

$\alpha_1, \ldots, \alpha_{2^n}$ list the $\underline{\text{atoms}}$ of L, ie sentences of the form

$$p_1^{\varepsilon_1} \wedge p_2^{\varepsilon_2} \wedge \ldots \wedge p_n^{\varepsilon_n}\ ,\ \ \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n \in \{0,1\}$$

Figure 1: layout of the blackboard

The fragment starts at the transition into the lecture, which is marked by the emphasised "okay" and the announced beginning, "let's begin". The lecture starts with a 'reminder' episode, which involves going over what has been written on the board and bounded by a line below. When uttering "remind you", the lecturer points to the text on the board, thereby projecting what he will remind the students. The lecturer adopts a 'starting position' with a step toward the board and a preparatory extension of the arm to bring the hand into place in an 'initial' position, close to the 'L' at the top left corner of the board.

The lecturer (lines 2-3) starts with a 'to be kept in mind' feature of L for the current proving work. 'L' stands for a language, but it is a finite one, and this needs to be 'kept in mind' since the proofs under development will not work for non-finite languages. It has an 'until further notice' character, i.e., *until* non-finite languages are explicitly introduced this feature remains in force.

2.1. Tracing/animating features

During this fragment, the lecturer frequently traces and 'animates' the symbols that have been written on the board. So for example, in line 3 when uttering "lets say p one, p two, up to p n" , the lecturer points first at p1, then at p2, and finally spreads his fingers relative to the dots on the board that extend the sequence to n. In other words, he 'functionally animates' the dots gesturally, indicating a start and end point – with 'bits' in between.

He then withdraws his hand to prepare for the next comment, namely about "a bit of notation" (line 4). This comment prefaces the return to reading what is on the board, while simultaneously tracing the symbols with his finger. So, for example, at the end of line 4 the finger first points to 'p' on the left of the equation sign, then to the 'exponent 1' of p, before finishing at the 'p' on the right of the equation sign.

Going over this new notation, prepares the way for commenting *upon* this new notation, i.e., commenting on a "nice property that this notation has" (lines 5-6). This is spelled out by tracing the equivalence relation that has been written in the top right corner of the board (lines 6-8). Mathematically, the 'nice property' consists in the possibility of moving from the typically used *symbols* for propositions (namely 'p' or 'not p') to *numbers* (1 or 0) representing these two possibilities. In effect, it is putting in place a powerful mechanism to 'talk' about propositions of the formal language.

2.2. Coordinating gestures, talk, and board

We can note the *anticipatory character* of the gestures, of the use of hands, and of the lecturer positioning himself with respect to the board. Furthermore, these are coordinated with the oral exposition, often in such a way that the positioning, the gesture and the identification of the relevant point in the written out proof are designed to coincide: in other words, as we get to a point in the oral reading out, the symbol is simultaneously pointed at with the hands.

So, for example, while uttering "nice property that this notation has" (line 6) the lecturer is moving from the left to the right board and extending his right hand toward the board – so that he is in a position to 'trace' the 'nice property' on the board (the equivalence relation). His finger nominates the 'V' and then moves to the 'p' and the 'epsilon' in the exponent.

'$V(p^{\varepsilon})$' is a 'self-standing expression', or a natural episode. We can note that having traced this expression, the lecturer withdraws his finger (beginning of line 7) to make a comment on the range of values for epsilon: "epsilon is either (.) zero or one". The withdrawal of his hand coincides with the length of the comment on epsilon. Withdrawing the hand momentarily also allows the 'repair' of the indexical "that" in "that's equal to one" which refers to the whole expression '$V(p^{\varepsilon})$' and not only to some aspect of it.

The coordination of body and hand movement with 'where we are in the exposition' can again be seen slightly later (middle of line 8): there is a 'done' or 'finished' gesture, marking a completion of the reading out. This being a 'reminder', the equivalence just talked about requires no further justificatory comment, because "we showed that last time" (line 8). The 'done' gesture initiates a move from the right to the left board, accompanied by an evaluation of the benefit or value of this equivalence relation, namely that it is a handy technique, "a useful little artefact" (lines 8-9).

2.3. 'Stepping back' (for an 'insertion')

Saying "a useful little artefact" is perhaps a kind of comment-in-passing, the possible utility to be noted but not further developed at this place in the topical sequence. Although it might also be that this artefact will be used later in the lecture. In either case, it is an 'insertion' or 'side sequence' into the development of the argument.

We want to note how the insertion of the comment leads the lecturer's positioning to turn around and glance toward that whole written out section of the board. This kind of 'stepping back' seems to happen when the lecturer makes such 'insertions'.

For example, he withdrew the hand in line 7, when inserting the range of values that epsilon could take ("either zero or one").

Similarly, later in the fragment (line 12), he moves away from the board when again remarking about the possible values that the exponent could take: "to some power (.) either zero (.) or one". There is a rhetorical "okay?" which presumably relates to the apparent and acceptable character of what has just been mathematically shown, and provides a completor for the point that had been under development.

In each of these cases, there is a momentary stepping back to complete the comment, before making a 'return move' to the board to continue the main argument.

2.4. Lists and units

We've been talking about the way in which the gestures go in close coordination with the recital of the formulae written on the board, integrating into them expanding observations or comments on the notability of features of that proof.

Now, however, we want to consider some of the gestures involved in respect to their *mathematical iconicity*. In other words, we look at gestures which represent the mathematical structure of the units under discussion.

In illustration, we point to the gestures accompanying the comments in line 10: "lets's (let) alpha one up to alpha two to the n list the so-called atoms". This involves the initial placement of the left hand at the beginning of the unit, and then the extension of the right hand to the end of that unit. This gestural display indicates to us the *list-like* character of the unit. In other words it displays one feature of 'atoms', namely that they can be written down as a list, from "one" up to "two to the n".

In line 11, the utterance "sentences of this form" is accompanied by a very similar gesture to the one accompanying "alpha one up to alpha two to the n". Again, the left hand is fixed at the beginning of the sequence; the right hand starts at a point close to the beginning of the sequence and then is extended to the end. However, we suggest that this operates more as a *'framing' gesture*, where the two hands in paused location indicate that it is the structure of the *unit* that is the object of attention.

The sense of this 'framing' is reinforced by the fact that immediately after the utterance, both hands are moved slightly away from the board, and then returned to accompany "so this sentence", retaining the 'framing structure' throughout, and thereby re-identifying the subject of the comment, which is the whole *unit*.

The lecturer then makes an additional comment upon the *form* of the atoms, namely that they have "each of the propositional variables in it" (lines 11-12). Here, we can see how different items on the board are being connected: the hand moves to the first propositional variable ('p1') at the bottom, then to the propositional variables at the top, before returning to the propositional variables at the bottom. In other words, first a finger is used to identify 'those things' in the conjunction, before the hand moves upwards to provide where these items have 'come from'.

Having provided a characterisation of the form of atoms, "each of the propositional variables in it to some power zero or one", the lecturer makes a point that might be less transparent, namely that "there are two to the n of these things" (line 13-14). The initial formulation of "alpha one up to alpha two to the n" proposed that the sequence of atoms extends to "two to the n". However, it has not yet been indicated why it should be so rather than, say, simply up to 'n'.

The next utterance provides an explication of this point. The lecturer again 'goes over' the sentence at the bottom. He moves his hand to the first exponent while uttering "cos you have two choices epsilon one", then moves to the second exponent, before retracting his hand slightly and then moving very quickly to the last exponent while uttering "and so on (.) up to (.) epsilon (.) n". This ending of the 'explanation' becomes: if you have n times 2 choices, there are two to the n choice in total.

3. Fragment Two

The next clip is taken from a different introductory course with a different lecturer. It contrasts with the first fragment, where the work was a matter of going rather smoothly over something that has been lectured on before. In this second clip, we are witnessing a first time through exercise of lecturing on a proof.

We have tried to capture the current state of the blackboard in the transcript (Appendix II). The board to the left is the main site for the work of the proof and has been separated into two 'pages'. The lecturer has written out, word by word, the beginning of this proof. The proof will proceed by considering three cases. However, in the process of going through the first case, the lecturer has had some difficulty in finding his way through the proof. He had taken a wrong turning and has erased the first draft of this part of the proof. He is now re-working this part of the proof.

3.1. General observations

This example shows more clearly than the first that the working out of the proof is a *directed activity*. In other words, current decisions about the construction of the proof in hand are oriented to what needs to be proved. In this case, what will have to be done is to transform one formal proof into another by considering three base cases.

The lecturer makes use of the whole space of the blackboard, using the left board to develop the (metamathematical) proof. He uses the right board to illustrate the 'object' of the proof, namely the transformation of one formal proof into another one. This allows the lecturer to give a clearer direction of 'where we want to get to' ("we gotta get to tha:t", line 8).

There is a sense here that the lecturer is himself having to work out each next step to take before writing it down. We can see the lecturer frequently checking against the board and against the sheet of lecture notes in his hand, where he is in the course of the proof and what goes next.

3.2. First replacement line

At the beginning of the fragment, the lecturer is starting to write what the 'replacement lines' in the new formal proof will be. The line on the board is only partially verbalised as "first we want that S pro:ves" accompanying writing out on the board that S proves t_i. This is then identified as "first we write tha:t". Then there is the beginning of an 'explanation' (">and that will be:") of *why* we are allowed to write that line; this utterance gets self-corrected by "that's what I said before" thereby making reference to what had been a correct but misplaced part of the proof as previously written up and erased.

We are still waiting for the "reason" of why it is permissible to write this line in the new formal proof. In lines 2 and 3, we are given *two* possible reasons, namely either t_i is a logical axiom or t_i is a nonlogical axiom.

We want to focus on how in lines 3 to 5 the lecturer uses his finger to visualise the two options for the first replacement line. The lecturer switches the sheet of notes from one hand to the other, and adjusts his posture to elaborate the point. The exposition makes visible that there are *two* possible cases, "either […] or:".

- In the exposition of the first case the lecturer first points to t_i (neglecting the S on the left hand side of the 'proves' symbol) before gesturing to 'LA' (beginning of line 4). The reason for this is that if t_i is a logical axiom, it does not matter what is on the left hand side of the formation: "in which case we can write it" (line 4).
- The second possibility is that t_i is "a member of S", i.e., a non-logical axiom. Any member of S can be (formally) 'shown' from S. In other words, if t_i is a member of S then "we can write it" (line 5). Note how this second case is gesturally animated (beginning of line 5): the t_i is almost literally taken out of S, moved 'over' the 'proves' symbol, and then placed on the right hand side.

3.3. Second replacement line

Having established the first line in the new formal proof raises the question of what to do next. The lecturer 'answers' this by reminding the students of "what we want" (line 6). He moves to the right board where the 'aim' or 'goal' is being sketched out.

The lecturer now writes out where they want to get to, namely that "S (-) proves (0.3) little s (0.2) implies t i". Having written down this 'aim', he returns to the left board and he repeats the current 'problem' (in lines 7-8): "we got tha:t" (the first line of the new proof), "we gotta get to tha:t" (the final line in the new formal proof, which has just been written on the right board).

So the next question seems to be: what intermediate steps will be necessary to get from here to there? In other words, whatever comes next is projected to provide a means of progression from where we are to where we want to get to.

This next line of the proof turns out to be "S pro::ves (0.5) t i (0.3) impli:es (0.2) s (-) implies t i" (transcript, lines 8-9).

The lecturer has already given an explanation of 'why' we want that line (namely to get from here to there). However, he has not yet explained (a) how exactly that line will do that job, and (b) why it is 'permissible' to write that line in the first place. He gives an answer to this second question: "because [...] that's a logical axiom" (lines 9-10).

There are a couple of self-corrections here, which we think exhibit a move from the particular to the general, and back to the particular. Logical axiom is the general form (all instances of the axiom schemes given in the lecture notes are logical axioms), whereas the number of an axiom scheme is the particular form. The problem for the lecturer: he can't remember the exact number of the axiom scheme.

All the axiom schemes are available in the notes. The lecturer provisionally identifies the number as "probably one" (line 11), but decides to check. He first looks at the sheet in his hand, and then (having not found the number there) at the complete set of lecture notes the table ("let me just check (0.3) in my list of axioms"). A student (very quietly) offers a candidate answer: "one", which the lecturer then confirms: "yeah (.) axiom schema one".

Now the explanation of why it is permissible to write this line is finally brought to a closure: "it's a logical axiom (.) its actually (.) axiom (0.3) scheme (0.2) one" (line 14).

The lecturer first writes "LA" (for logical axiom") *and* "Ax (1)" (for axiom scheme one), but then deletes the "LA" (beginning of line 15). This self-correction is not a matter of correcting an 'error' in the proof progression but rather a rethinking of the *specificity* of what needs to be said: "an instance of an axiom scheme" is a more precise provision for the recognisability of the line's efficacy.

The lecturer then checks in his notes to find that this is an instance of axiom scheme one.

3.4. Third replacement line in the proof

Having found the reason for being able to write the second line, the lecturer is now in the position to resume developing this formal proof. In fact, it turns out that he is able not just to 'resume' but to bring the formal proof to a finish.

"And now::" (line 15), accompanied by a return to the board and a downward hand stress, announces the conclusion of the deductive sequence. It prefaces "you can write (.) what we want" (line 16), where the indexical 'what we want' is made sense of by pointing to the right board depicting the 'goal' of the current project.

So the lecturer's question-to-answer is: What enables us, having only established these two lines, to get where we want to get to?

The answer is immediately provided: "because by modus ponens" (line 17). Modus ponens is the utterly familiar and basic form of formal logical reasoning (which most of you might know in the form of "Socrates is human. Every human is mortal. Hence Socrates is mortal."). The modus ponens has a special status here, as it is, within the 'game' of 'formal proofs', the only move that allows you to combine two lines to a create a new line.

Modus ponens, in a sense, needs two 'input' lines and gives one 'output' line. And this seems to be 'animated' gesturally while the lecturer says "because by modus ponens" (line 17): he first points at the first line, then at the second line, and then moves his hand slightly away from the board, before arriving at the place where the third line will appear. Note the anticipatory swivel move of the hand before placing it in writing position. There is a sense of "one (.) two (.) gives (.) three" which says what the next line is going to be, namely, the last step in the format.

This generalised form of modus ponens is then subsequently particularised (line 18): "we have that" (pointing at t_i in the first line), "we have that" (pointing at $t_i \rightarrow (s \rightarrow t_i)$ in the second line), "implies that" (pointing at $s \rightarrow t_i$ in the second line). This is, in a sense, another way of 'visualising' modus ponens. Whereas the first way displayed the structure as three lines, this visualises the structure of modus ponens *within* the two lines already written. The finger identifies the elements of the modus ponens deduction *in* the first and second line, and then shows the 'result' of applying modus ponens to these elements at the end of the second line.

4. Conclusion

In this initial examination of our data we have identified four main ways in which the lecturer's 'handiwork' features in the exposition proofs. These are:

- Firstly, that of connecting what is being written on the board with the commentary that accompanies it.
- Secondly, the way in which gestures 'integrate' the different *sections* of the board. The board is divided up into different sites – effectively 'pages' – and sometimes two or more of these pages may be in concurrent development, and there are gestures which direct student attention from the instant point of the work toward relevant (places on) other pages which consequentially relate to that point of development.
- Thirdly, the way in which gestures 'integrate' the *organisation* of the proof. In other words, gestures connect mathematical points across the order of expositions, for example, (a) to indicate a relationship between one element and another (that this point is the same as that made earlier) or (b) to locate the license provided by an earlier entry for a current move.
- Finally, there are those gestures which we have called 'mathematically iconic', which visualise properties of a mathematical feature, such as visualising the working of 't_i in S' or 'modus ponens'.

Our remarks have been threaded through with thoughts about the 'task structure' in the further extension of developing proof sequences from moments within that structure, though we have mostly done so incidentally. Thus, we have touched on the fact that current steps in the operation are understood as working toward some required appropriate achievement, that seeing what the achievement must/will be is an assist to understanding what is being done now and needs to be done next in order to get there. We have also only touched upon the ways in which the ordered properties of the mathematical apparatus are used to do the work of further developing the proof, how their 'placement' within the proof sequence moves the sequence towards its objectives, and provides the materials out of which further moves can be constructed. Finally, we have rather presupposed the way in which the current work is embedded in pre-existing mathematical understandings which enable recognition of the adequacy of proposed steps in the sequence to the task of building a coherent structure. Much more intensive exploration of these features is necessary for an adequate characterisation of the mathematical reasoning embodied in the lecture's course.

5. Acknowledgements

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6. References

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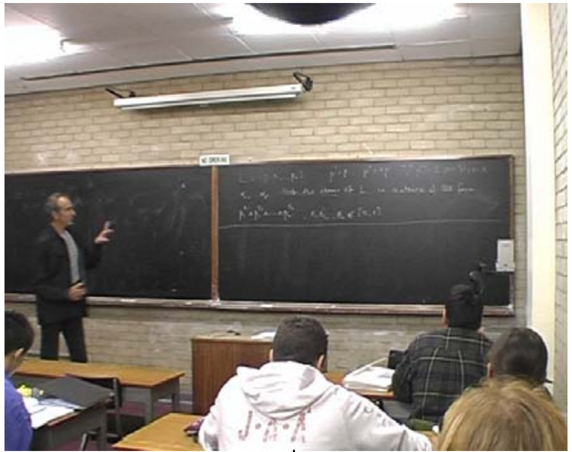
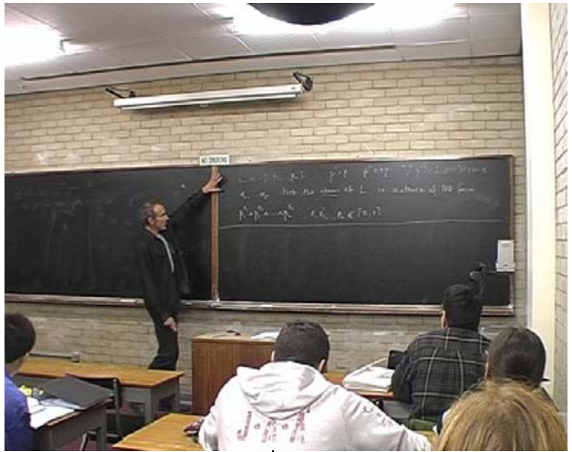
7. Appendix 1

01: >okay: (-) let’s begin





points to text on board

moves towards board

02 (--) uh:: (-) remind you what happened last time (-) :h for this section we’re

points at p1 to pn

03 certainly working in a finite language L (-) let’s say p one, p two, up to p n (-) we

follows ‘notation’ with finger

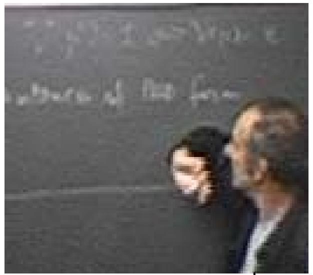
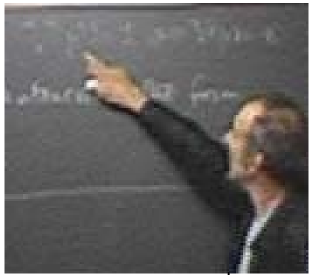
04 introduced a bit of notation (---) p: to the one is jus gonna be p (-) °if p is a

moves from left to right of board

05 propositional variable (-) p to the nought is not p (-) :h and:: (-) nice property

traces left hand side of equivalence

06 that this notation has is >you take a valuation V (-) then V: of p to the epsilon


moves hand away from board finishes tracing left hand side traces rest of equivalence

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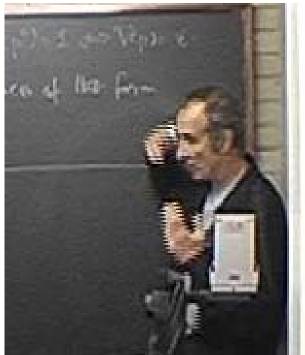
07 where epsilon is either (-) zero or one (-) that’s equal to one (-) if and only if V



traces rest of equivalence ‘done’ gesture moves from right to left of board

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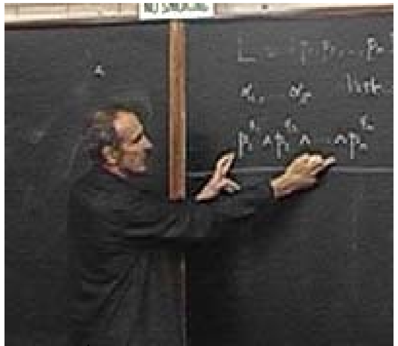
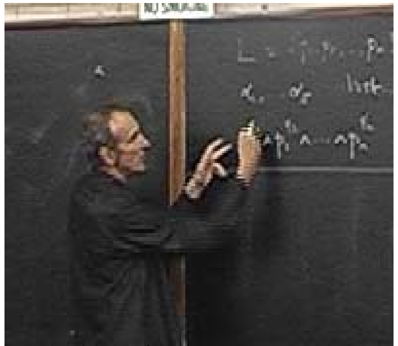
08 of p: is equal to epsilon (-) we showed that last time (-) >sothat’s a useful little-

09 (----+----+----+) uh::: (----+----+----+----+) artefa- >artefact (---) >artifice (-)

pointing at α one to α two to the n

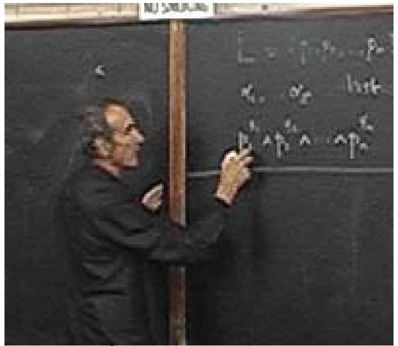
10 >now (---) let's (let) alpha one up to alpha two to the n list the socalled atoms

retracts hands

pointing at p one to p n points again moves to top via exponent

11 of L (-) thatis the sentences of this form (-) >so this sentence has each of the



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           moves hand down again     moves body away from board           moves body back to board
----------.--------------------.-----------------.-----------------.-------------------
12  propositional variables in it (-) to some power (-) either zero (-) or one (-) °okay?
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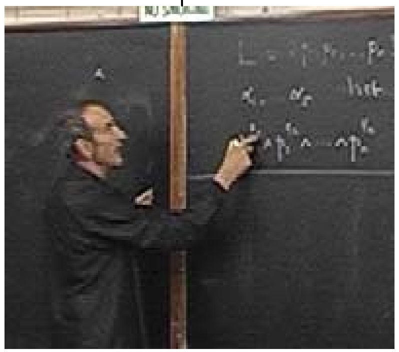
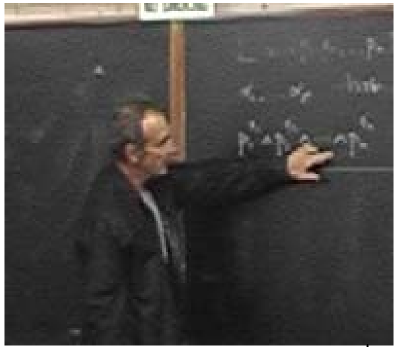
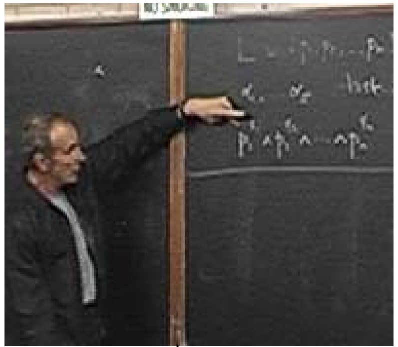
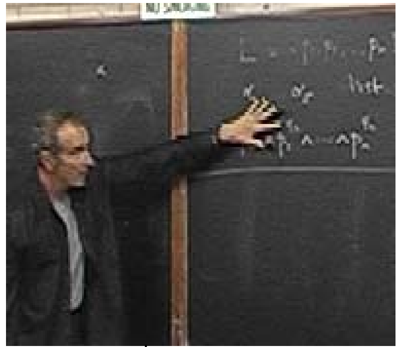




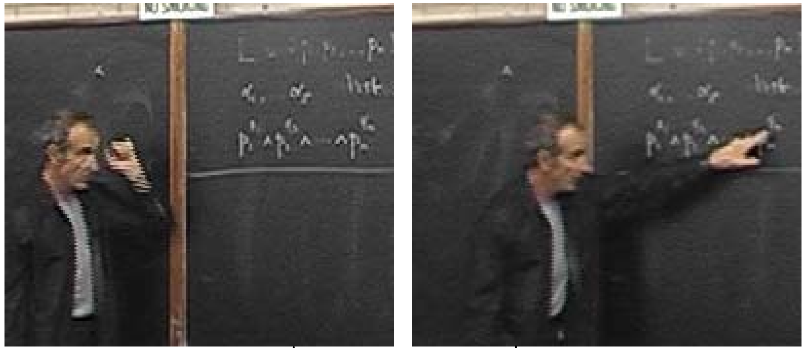
points at conjunction   points at top   moves to index

13 (-) the conj:unction of those things (-) so in particular there are two to the n (-)



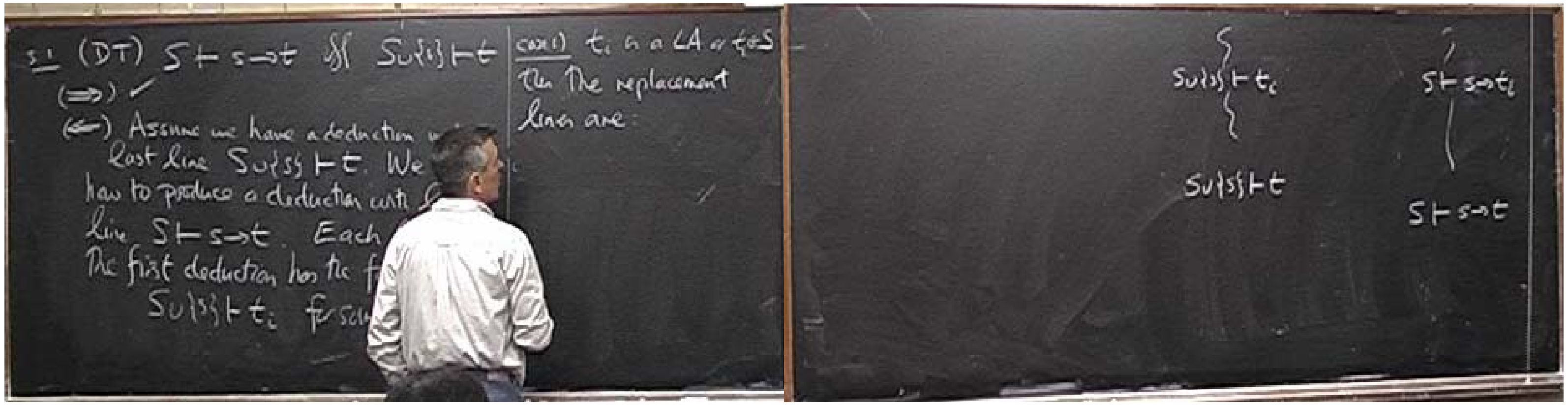


14 of these things (-) °cos you have two choices for epsilon one (-) two choices

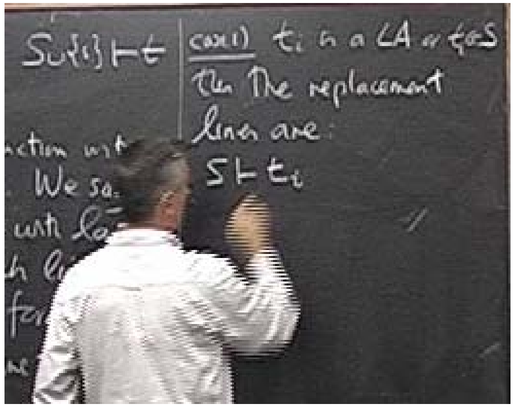


for epsilon two and so on (-) up to (-) epsilon (-) n: (0.3) and let’s let

8. Appendix II

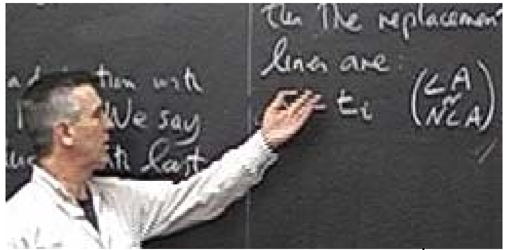


[start of the fragment (reconstructed)]



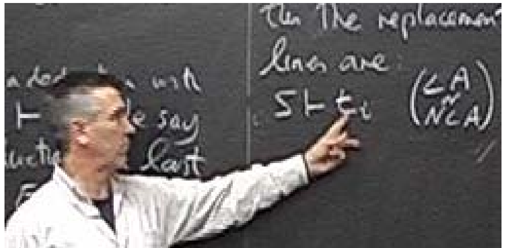
01 <u>so::</u> (0.2) first we want that S pro:ves: (1.5) first we write tha:t (2.0) >and that

02 will be: (-) >that's forthe- (-) that's what I said befo:re (-) for the reason that



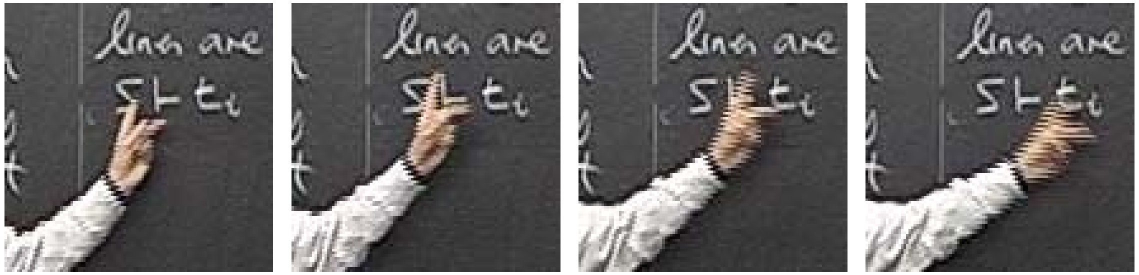
writes LA or NLA points at t i

03 (-) it's a logical axiom (0.5) or: (-) nonlogical axiom (3.0) in this case either t i



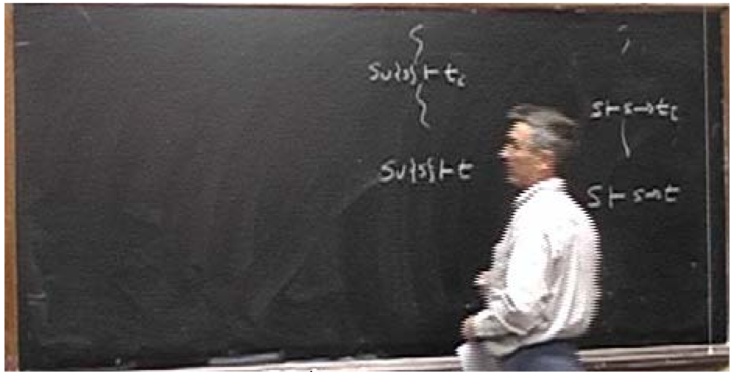
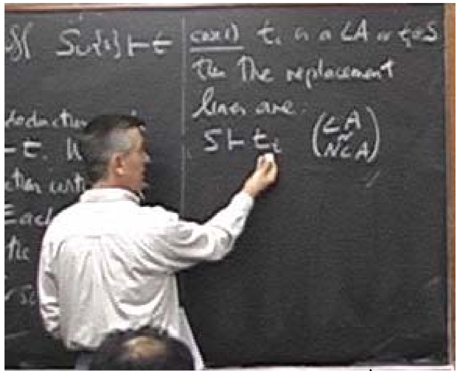
points at LA (on right) to t i stays on t i

04 is one of those axioms (-) in which case we can write it (-) or: it's a member of S



moves over ‘|-‘
.-------------.   .----------
05 (-) in which again- >this again (-) we can write it (2.5) okay? (----+----+----+----+)

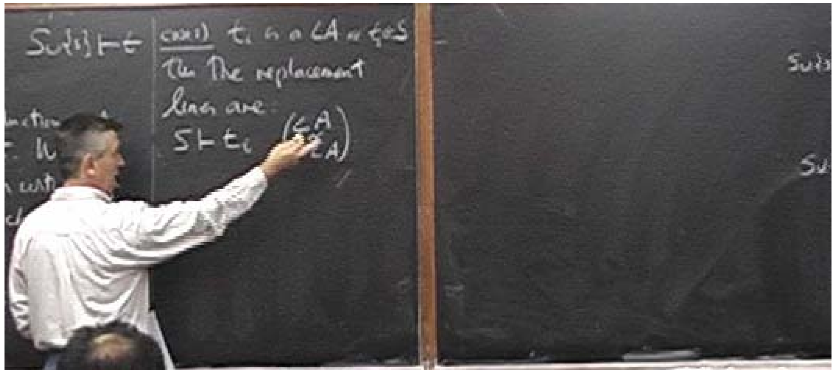
moves to the board on the right
--.
06 >thewh- >what we want: (0.5) is to get: (---) S (-) pro:ves (0.3) little s (0.2)

points at first line

moves back to the board on the left

```
   .----------------------------------.
07  implies t i (----+----+----+----+----+----+----+) okay: (-) we got tha:t (-) we
```



points at 'goal' on the right

```
08  gotta get to tha:t (-) so what we (-) next write is (-) S pro::ves (0.5) t i (0.3)
```

09 impli:es (0.2) s (-) implies t i (2.0) because that's a (??) instance of ax- >that's a

10 logical axiom (-) that's an instance of axiom schema:::

looks at paper in his hand looks at script on table

..-----------------------------------...-------------------------------------

11 (---) >probably one (----+----+----+----+----+----+----+----+----+----+----+)

----

12 (----+----+----+----+----+----+----+----) let me just check (---) in my list of

moves back to board

..------------------------.

13 axioms (----+---)>yeah (-) axiom schema one (----+----+----+----+)

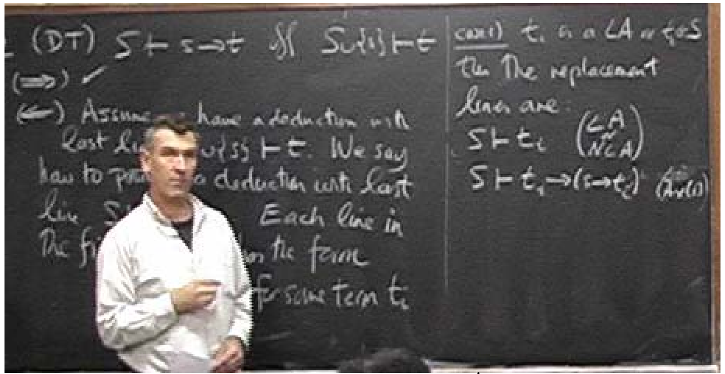
[

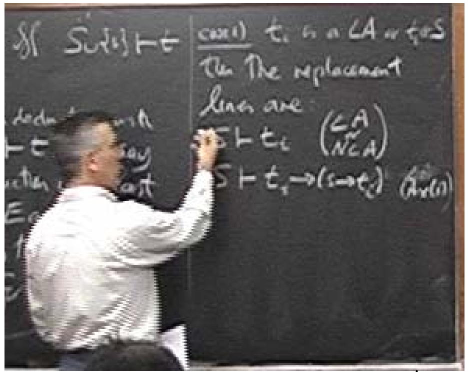
S: °one

writes "LA Ax (1)"

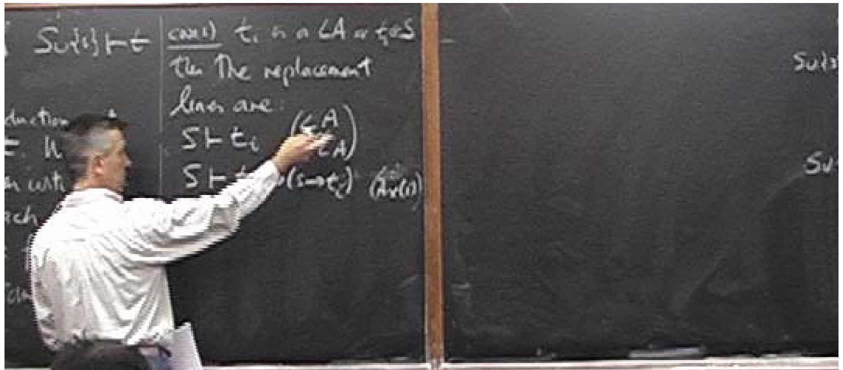
.---.

14 >so this is a logical axiom (-) it's actually (.) axiom (0.3) scheme (0.2) one

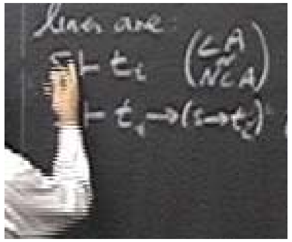
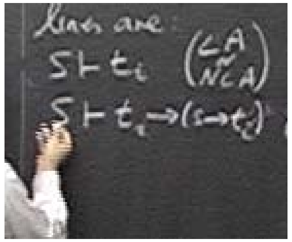
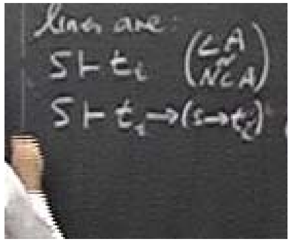
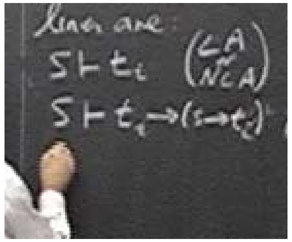




```
 deletes "LA"      moves to left, facing class                        moves back to board
   --            .-----------------.                               ..--------------------.
15  (----+----+----+----+) right? (----+----+----+----+) >and now:: (-) you
```



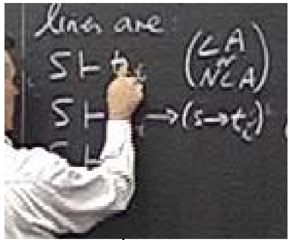
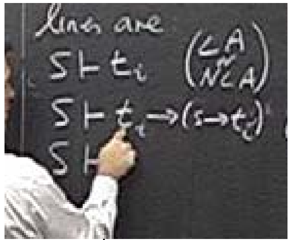
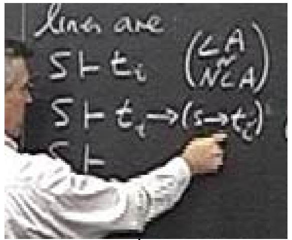
```
    points to right
    .--------.
16  can write (-) what we want (-)
```

‘modus ponens’

.----------------.

17 because by modus ponens (0.2) >so S pro:ves:

18 (---) we have that (-) we have that (-) implies that (-) (inaudible) this (0.5)

19 >small s (0.5) by modus ponens